B2 The 5-Second Pendulum

Key Question: *What length of string would produce a 5-second pendulum?*

In this investigation, students will use data collected from Investigation B1 to come up with an equation to calculate period from string length. Many students are amazed that math can be used to predict actual experimental results! They will solve their equations for string length and then extrapolate the string length required for a 5-second pendulum.

Learning Goals

✔ State a hypothesis that describes how string length and period are related.

✔ Graph the hypothesized relationship and use the graph to derive an equation for determining the period given the string length.

✔ Use the equation to predict the string length needed to create a 5-second pendulum.

GETTING STARTED

Time 50 minutes

Setup and Materials

1. Make copies of investigation sheets for students.
2. Review all safety procedures with students.

Materials for each group

- A copy of string length vs. period data and graph from Investigation B1
- A graphing calculator or software capable of curve fitting*
- Graph paper*

**provided by the teacher*

Online Resources

Available at curiosityplace.com

- Skill and Practice Sheets
- Whiteboard Resources
- Animation: Pendulum Graph
- Science Content Video: Frequency
- Student Reading: Harmonic Motion Graphs

Vocabulary

best fit curve – a curved line that fits the points on a graph and connects them into a smooth curve

extrapolation – using data to find an equation that can be used to predict values outside the range

function – a mathematical equation that shows the relationship between two variables

graph – a visual representation of data

inverse relationship – a relationship in which one variable decreases when another variable increases

period – the time it takes for one cycle

NGSS Connection

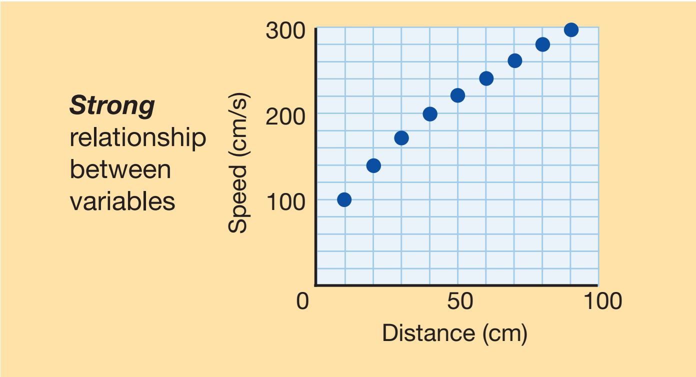
This investigation builds conceptual understanding and skills for the following performance expectation.

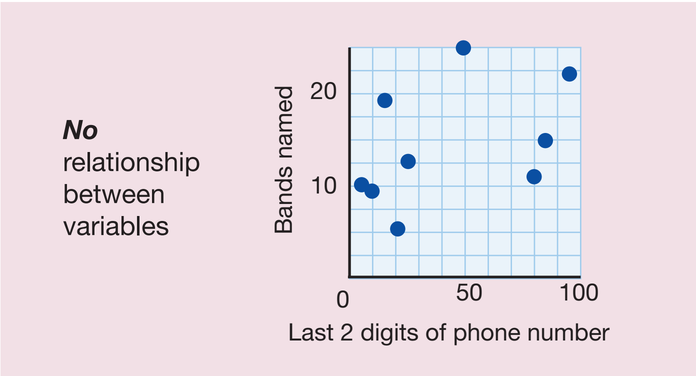
HS-PS4-1. *Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.*

Science and Engineering Practices	Disciplinary Core Ideas	Crosscutting Concepts
Using Mathematics and Computational Thinking	PS4.A: Wave Properties	Cause and Effect

BACKGROUND

In many experiments, you are looking for a cause-and-effect relationship. How does changing one variable affect another? **Graphs** are a simple way to see whether there is a connection between two variables. With a graph, you can see connections that you might miss from simply looking at tables of data. When there is a relationship between the variables, a graph shows a clear pattern. Using the pattern, you can often identify a **best fit curve**, or trend line. For example, the speed and distance variables show a strong relationship in the top graph below. When there is no relationship, the graph looks like a collection of dots, as in the bottom graph. No pattern appears. The number of rock bands a student can name in one minute and the last two digits of the student's phone number are two variables that are not related.





An **inverse relationship** exists when one variable increases and the other decreases. If you graph how much money you spend against how much you have left, you see an inverse relationship. The more you spend, the less you have. Graphs of inverse relationships often slope down to the right.

Extrapolation is a process that is used to estimate quantities outside the range of your data that cannot be easily measured. Data is collected for a small number of points, and then that data is used to determine the relationship between the variables. A **function** is a mathematical equation that shows the relationship between two variables. Once the relationship is known, it can be used to determine the value of one of the variables based on a value of the other variable. The relationship between the dependent and independent variable can be expressed in a graph, or mathematically as an equation or formula.

PERIOD OF SMALL AMPLITUDE PENDULUM

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Period (s)
Length (m)
Acceleration due to gravity (m/s^2)

The formula for the **period** of a small-amplitude pendulum is shown above. For amplitudes less than 20 degrees, a pendulum's period is nearly constant and only depends on its length and the acceleration due to gravity. The difference between the period of a 1-degree pendulum and that of a 20-degree pendulum is less than 1 percent. When a pendulum's amplitude exceeds 20 degrees, however, its period differs from that of a small-amplitude pendulum by a more noticeable amount of up to a few percent. This difference comes from the use of the small angle approximation $\sin\theta \approx \theta$ in the derivation of the formula for the period.

5E LESSON PLAN

Engage

To activate and assess prior knowledge, ask students how they can tell if two variables in an experiment are related. Students performed an experiment in the previous investigation that showed a relationship between the pendulum's period and length of string. How did they know that there was a relationship? In what ways could they show that those two variables are dependent on each other?

Allow students time to brainstorm answers to these questions. Discuss the answers as a class. Use guiding questions to help students explore the problem. Point out the variation in student answers. Accept all answers.

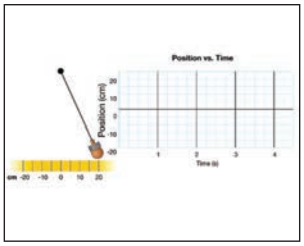
Explore

Have students complete Investigation B2, *The 5-Second Pendulum*. Students use their data from the previous investigation to come up with an equation to calculate period from string length. They solve their equation for string length and then extrapolate the string length required for a 5-second pendulum. To complete their analysis, they compare their equation with Huygens's derived equation for pendulum period.

Explain

Revisit the Key Question to give students an opportunity to reflect on their learning experience and verbalize understandings about the science concepts explored in the investigation. Curiosityplace.com resources, including student readings, videos, animations, and whiteboard resources, as well as readings from your current science textbook, are other tools to facilitate student communication about new ideas.

Science Content Video
Frequency



Animation
Pendulum Graph

Elaborate

In the last part of the investigation, students learn the formula for the relationship between the period of a pendulum and the length of its string.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where: T is the period, l is the length, g is the acceleration of gravity.

Take a closer look at the relationship between the acceleration due to gravity and the period of the pendulum. Have students hold the string length constant, and examine the change in period when gravity changes. Graph the formula for a range of values of g. Analyze the shape of the graph—is it linear or curved, direct or inverse?

How would a pendulum on a different planet differ from the same pendulum on Earth? Have students use their graphs to find out. Values of g for different planets are listed below:

Mercury	3.61
Venus	8.83
Mars	3.75
Jupiter	26.0
Saturn	11.2
Uranus	10.5
Neptune	13.3

Evaluate

- During the investigation, use the checkpoint ✔ questions as opportunities for ongoing assessment.
- After completing the investigation, have students answer the assessment questions on the *Evaluate* student sheet to check understanding of the concepts presented.

Explore INVESTIGATION **B2**

Name ______________________ Date ______________

B2 The 5-Second Pendulum

What length of string would produce a 5-second pendulum?

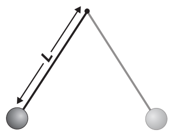
Materials:

- ✔ A copy of string length vs. period data and graph from Investigation B1
- ✔ A graphing calculator or software capable of curve fitting
- ✔ Graph paper

What length of string would you need to build a pendulum with a **period** of 5 seconds? You can't simply read the **graph** of string length vs. period that you created in the previous investigation, because the graph doesn't extend to 5 seconds. To complicate matters further, the graph is not a straight line, so it is not easy to predict what it would look like if it did extend to 5 seconds.

The solution is to model an equation to fit your data. Then you can use the equation to predict the period of the pendulum from the length of the string. Then, using the same equation, you could determine string lengths or periods that were not on your original graph. Using data to find an equation to predict new values outside the range of your data is called **extrapolation**.

In this investigation, you will use your data to find an equation that tells you the period if you know the length of a pendulum. You will then use your equation to predict the string length that would produce a 5-second pendulum.



❶ Recognizing patterns from a graph

First, you'll be taking a look at the graph of your data from the previous investigation. Observing the shape of the graph will help you recognize patterns in the data and develop the equation that models the data. Use the graph to answer the following questions.

a. What is the shape of the curve on your graph? Does it curve closer toward the axes or does it curve away from the axes? Is it shaped more like a cup or a bell or a banana? In which direction does the curve face?

It has a single curve that curves away from both axes. It looks like a banana, but has the "cup" shape facing down.

b. What can you say about the relationship between the variables? Is it a direct relationship or an **inverse relationship**?

We can see from the graph that the period increases as the string length increases. That makes it a direct relationship.

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Guiding the INVESTIGATION

❶ Recognizing patterns from a graph

Students may have trouble analyzing a graphical relationship shown by a curved line. The curve makes it difficult to determine the mathematical relationship between length and period. Help them to describe the curve on the graph. Encourage them to categorize the curve just by looking at it.

Ask leading questions to help with their descriptions:

- When the length increases, what happens to the period?
- Does the period change in the same proportion as the length? For example, when the length is doubled, does the period double?
- Is it a strong relationship?
- What does the shape look like?
- Does the curve show an inverse or direct relationship?

TEACHING TIP

As an optional activity, help students build the 5-second pendulum. Be sure to choose a safe place for the students to do this, such as a stairwell, a gymnasium, or a stage. Work with your school's facilities management to locate a space at least 7 meters high with a place to affix the pendulum.

Materials required:

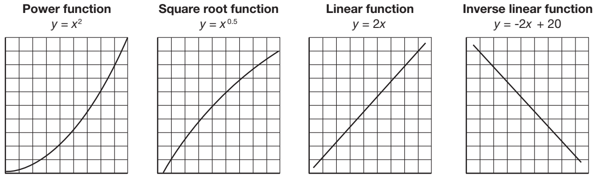
- One plastic half-gallon jug filled with water (to use as a bob)
- 7 meters of lightweight nylon cord or braided nylon rope (1/8" diameter or less)
- Stopwatch

Students should measure the length of the pendulum from the pivot to the center of mass of the jug. When timing the period, they should find that the error in measurement should be within about 2 percent.

Don't start the pendulum with a large push. Small amplitudes are more accurate for period measurements, and large amplitudes could be dangerous. Keep all students clear of the pendulum bob, and watch out for windows.

Explore — INVESTIGATION B2

c. Which of the following types of graphs does your graph look most like? This will help you determine what kind of equation you'll use to model your own graph.



The best match for my graph is the square root graph.

❷ Recognizing relationships in data

Table 1 will help you organize the data you collected in Investigation B1. Copy the string length vs. period data from B1 into the first two columns of Table 1. List the data from smallest to largest values.

Fill in the last three columns of Table 1. Calculate the experimental values for the three relationships: T/l, T^2/l, and T^3/l, where T is the value for period and l is the value for string length. These calculations will help you discover the relationship between the variables. The first row provides sample calculations to help you.

Table 1: Length and period data

String length (m) l	Period (s) T	T/l	T^2/l	T^3/l
0.2	0.8734	4.367	3.814138	0.762828
0.4	1.2524	3.131	3.921264	1.568506
0.5	1.4054	2.8108	3.950298	1.975149
0.6	1.541	2.568333	3.957802	2.374681
0.8	1.7778	2.22225	3.950716	3.160573

Sample data shown for a pendulum with 8 washers at 10 degrees amplitude.

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Explore — INVESTIGATION B2

a. What do you notice about the calculated values in the last three columns in Table 1?

The T/l column and the T^3/l column have a wide range of values. The calculations in the T^2/l column are all close together.

b. What does this tell you about the relationship of the string length to the period?

The period squared divided by string length equals a constant.

✔**c.** Write a hypothesis for how the period of the pendulum relates to its string length.

The square of the period is proportional to the string length, or, looking at it in terms of period, the period is proportional to the square root of the string length.

❸ Determining a mathematical relationship

You have examined your data set from Investigation B1 and its graph closely, and made a hypothesis about how the variables are related. Now you'll use technology to determine the **best fit curve** that fits your data. A best fit curve is a curved line that fits the points on your graph and connects them into a smooth curve. The best fit curve will give you the **function** for your data set. A function is a mathematical equation that shows the relationship between two variables.

1. Use the technology (graphing calculator or software) provided by your teacher to enter your data set.
2. Once your data set is entered, it is time to select a type of function to fit the best fit curve to your data. Some examples of function type are linear, power, square root, or polynomial. Use your hypothesis from Part 2 to select the function type.

   a. Find the curve that best fits your data. Then, determine the function for the curve. This expression will be in the form of an equation for the period of the pendulum. This equation is a mathematical statement of your hypothesis. Write your equation below. Use T for the period value, and l for the value for string length.

   $T = 2.001\ l^{0.5}$ or $T = 2.001\ \sqrt{l}$

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Guiding the INVESTIGATION

❸ Determining a mathematical relationship

Students will be finding an appropriate equation for the data using graphing calculators or other data software. This will give students practice with using data analysis skills and developing mathematical models for their investigations.

The previous parts of the investigation have helped students predict the type of function that fits the trend line for their data set. They will have seen the types of graphical shapes for different functions and performed some data manipulation (linear, squared, cubed) to narrow down the relationship between string length and period. Use the results of these exercises to guide students toward understanding that the string length increases with the square of the period.

STEM CONNECTION

Scientists in the 1800s used pendulums to precisely measure the intensity of gravity. By observing a carefully calibrated pendulum's period, the acceleration due to gravity could be determined. Changes in the period indicate changes in the gravitational constant. Determining the local gravitational constant in many different locations around the world helped scientists map out Earth's gravity field. The research concluded that gravity is not constant over all of Earth, but instead varies. The results of this research helped scientists establish that the physical shape of Earth is not a perfect sphere, but is instead an "oblate" shape with a bulge in the center along the equator.

Explore — INVESTIGATION B2

❹ Using the equation to find string length for a 5-second pendulum

Use your equation to extrapolate the length of string needed to make a 5-second pendulum.

a. Solve your equation for the length of the string (l). Use this version of your equation to calculate the string length needed to make a 5-second pendulum.

$$l = \frac{T^2}{4.004}$$

b. Predicted string length for 5-second pendulum: 6.24 meters

❺ How does your equation compare to the derived equation?

In the 1600s, Christiaan Huygens (1629-1695) derived an equation for the relationship between the period of a pendulum and the length of its string. His equation is valid for small angles of amplitude.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where: T is the period, l is the length, g is the acceleration of gravity.

a. What is the solution of Huygens's equation if you use the values of $g = 9.8\ \text{m/s}^2$ and $\pi = 3.14$?

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad T = (2\times 3.14)\sqrt{\frac{l}{9.8\ \text{m/s}^2}} \qquad T = 2.006\sqrt{l}$$

b. How does this compare to your equation?

Our equation is very close, within a few thousandths for the constant.

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Evaluate — INVESTIGATION B2

Name ______________________ Date ______________

1. In the investigation, you found a relationship between string length and period of the pendulum. Finding mathematical relationships between variables is a very important aspect of scientific research. See if you can find the relationships between the x and y variables in the data sets below. The first one is done for you.

Data set 1		Data set 2		Data set 3	
x	y	x	y	x	y
1	2	1	1	1	1
2	5	2	8	2	1.41
3	10	3	27	3	1.73
4	17	4	64	4	2
5	26	5	125	5	2.24
$y = x^2 + 1$		y = ______		y = ______	

Data set 2: $y = x^3$ Data set 3: $y = \sqrt{x}$

2. Use your equation from Part 3 to extrapolate the length of string needed for:

   a. a 3-second pendulum

   3-second pendulum: 2.24 meters

   b. an 8-second pendulum

   8-second pendulum: 15.98 meters

3. The Moon's value for g, the acceleration due to gravity, is $1.62\ \text{m/s}^2$. Use Christiaan Huygens's formula to calculate the length of a 5-second pendulum on the Moon.

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad l = g\left(\frac{T}{2\pi}\right)^2 = 1.62\left(\frac{5}{2\pi}\right)^2 \qquad l = 1.02\ \text{meters}$$

4. Would a pendulum have the same period at sea level as it does on top of Mt. Everest (8,848 meters)? *Hint*: Think about the acceleration due to gravity.

   The pendulum would have a slightly longer period on top of Mt. Everest, because the acceleration due to gravity is a little less than at sea level.

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WRAPPING UP

Have your students reflect on what they learned from the investigation by answering the following questions:

1. How can we use our string length vs. period data to predict a value outside the data?
2. How can an equation be determined from the graph of a line?