B3 Relating Launch Speed and Range

Key Question: *What function relates the range and launch speed of a projectile?*

In this investigation, students identify the function relating projectile range to launch speed. In doing so, students are exposed to a variety of graphical relationships. They must choose the relationship that most closely matches that of the data they gathered in Investigation B2.

Learning Goals

- ✔ Prepare graphs relating range and launch speed using different functions of launch speed.
- ✔ Determine the relationship between range and launch speed using the results of the graphs.

GETTING STARTED

Time 50 minutes

Setup and Materials

1. Make copies of investigation sheets for students.
2. Watch the equipment video.
3. Review all safety procedures with students.
4. Have students work in small groups of three to five.

Materials for each group

- Data from Investigation B2, *Launch Speed and Range**
- Calculator*

**provided by the teacher*

Online Resources

Available at curiosityplace.com

- Equipment Video: Marble Launcher
- Skill and Practice Sheets
- Whiteboard Resources
- Animation: Average and Instantaneous Speed
- Science Content Video: Average and Instantaneous Speed
- Student Reading: Speed

Vocabulary

function – a mathematical operation that relates two or more variables

NGSS Connection

This investigation builds conceptual understanding and skills for the following performance expectation.

HS-PS2-2. *Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.*

Science and Engineering Practices	Disciplinary Core Ideas	Crosscutting Concepts
Using Mathematics and Computational Thinking	PS2.A: Forces and Motion	Systems and System Models

BACKGROUND

This investigation does not require any new data collection but instead involves a more in-depth analysis of the data from B2, *Launch Speed and Range*. Prior to beginning the investigation, review with students the meaning of the mathematical term **function** and the relationship between independent and dependent variables. When studying the effect of launch speed on a projectile's range, the speed is the independent variable because it is the one that is intentionally changed. The range is the dependent variable; its value is determined by the value of the launch speed. When displaying data on a graph, the dependent variable is graphed on the *y*-axis, and the independent variable is graphed on the *x*-axis.

A function is a relationship between two sets of numbers that explains how one set is related to the other. Each value in one set maps to exactly one value in the other set. On a graph, this means that each x-value corresponds to exactly one y-value. A "vertical line test" is used to determine if a graph is a function. A vertical line drawn anywhere on the graph of a function passes through exactly one point on the curve. Several types of functions are described below:

Type	Description	Mathematical notation
linear (or direct)	y is proportional to x	$y \propto x$
quadratic	y is proportional to x^2	$y \propto x^2$
inverse	y is proportional to 1/x	$y \propto 1/x$
square root	y is proportional to the square root of x	$y \propto \sqrt{x}$

Looking at the shape of a graph of *y* vs. *x* is helpful in determining the function that relates the data. In this investigation, students should see that their graphs of range vs. launch speed resemble the graph for $y \propto x^2$; their curves are the same shape. The function can be checked by graphing range on the *y*-axis and speed^2 on the *x*-axis. This graph should yield a straight line, indicating that range is proportional to the square of launch speed.

Advanced Math Background – Ratios and proportionality

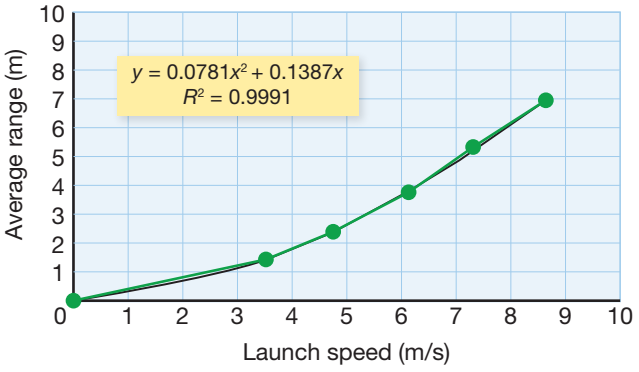
In this investigation, students examine experimental data and look for proportional relationships. Students may confuse proportional relationships with a function like $y = mx + b$. In the function $y = mx + b$, y varies proportionally with x, and in fact when solved with the y-intercept value b, this equation will determine the value of y. Meanwhile, we could say that for the same value m, $y = mx$ is the same proportional relationship as our original equation in $y = mx + b$. However, solving for y using $y = mx$ will not give the same value as $y = mx + b$. In a similar fashion, the procedures for looking at proportional relationships in Part 1 will show proportionality between the values of range and launch speed. However, these proportional relationships are not predictive in the sense that $y \propto v^2$, $R \propto v^2$, $y \propto x^2$ is in fact $y = x^2$. There are variables at work in the physical relationship between launch speed and range that can only be captured with a more complex formula.

For example, review this sample data from Investigation B2 (see facing page), the graph of the data, and the trend line fitted to the data. (*Note:* To better approximate the mathematical relationship, the data were extrapolated by adding the additional data point of 0, 0).

B2 Data with 0, 0 superimposed on data

Spring setting	Average time (s)	Launch speed (m/s)	Average range (m)
0	0	0	0
1	0.0054	3.52	1.43
2	0.004	4.75	2.39
3	0.0031	6.13	3.76
4	0.0026	7.31	5.33
5	0.0022	8.64	6.95

Graph of B2 Data with 0, 0 superimposed on data



This trend line was developed using statistical methods in a spreadsheet program. As you can see, the line fits the data well. However, there are additional terms in the equation of the line, the coefficient 0.0781 in the term $0.0781x^2$ and the additional term $0.1387x$.

Now, let's look at a graph of the experimental data and a line derived from the trend line superimposed on that graph. Also, we'll include a line derived from an equation $y = x^2$ which is what students would have found if they mistakenly thought that the expression proportional to the function of the data was in fact a function of the data.

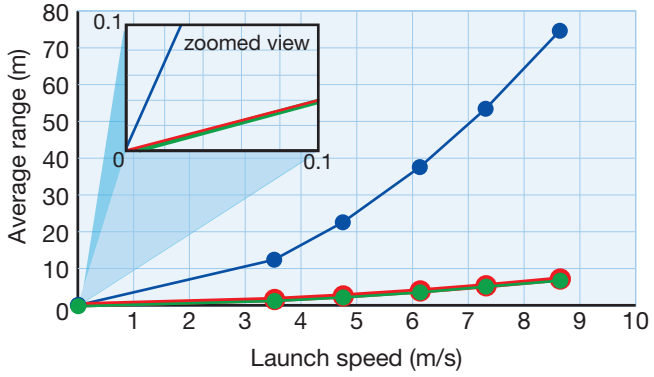
B2 Data derived from trend line equation, $y = 0.0781x^2 + 0.1387x$

Spring setting	Average time (s)	Launch speed (m/s)	Average range (m)
0	0	0	0
1	0.0054	3.52	1.455914
2	0.004	4.75	2.420956
3	0.0031	6.13	3.784987
4	0.0026	7.31	5.187256
5	0.0022	8.64	7.028502

B2 Data derived from proportional equation, $y = x^2$

Spring setting	Average time (s)	Launch speed (m/s)	Average range (m)
0	0	0	0
1	0.0054	3.52	12.3904
2	0.004	4.75	22.5625
3	0.0031	6.13	37.5769
4	0.0026	7.31	53.4361
5	0.0022	8.64	74.6496

Graph of B2 Data with B2 derived and B2 trend line



B2 Data derived from proportional equation, $y = x^2$

B2 Data derived from trend line equation, $y = 0.0781x^2 + 0.1387x$

B2 Data with 0, 0 superimposed on data

As you can see, the line representing *B2 Data derived from proportional equation, $y = x^2$* deviates abruptly from the experimental data. Meanwhile, the line of *B2 Data derived from trend line equation*, $y = 0.0781x^2 + 0.1387x$ fits the data so well that it covers the line of *B2 Data with 0, 0 superimposed on data* in the graph.

5E LESSON PLAN

Engage

Investigation B3 uses functions. What do your students know about functions? Use the following activity to find out:

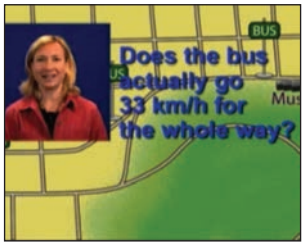
1. Ask students, "What is a function in mathematics?" Have them write brief answers, including examples, in their science notebooks.
2. Have students share their answers with a partner. Let them know that they may include anything they like or agree with in their own response.
3. Discuss the responses as a class.

Explore

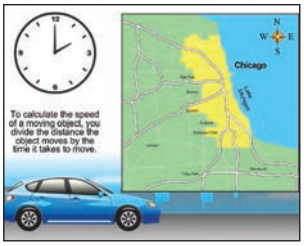
Have students complete Investigation B3, *Relating Launch Speed and Range*. In this investigation, students identify the function relating projectile range to launch speed. In doing so, students are exposed to a variety of graphical relationships. They must choose the relationship that most closely matches that of the data they gathered in Investigation B2, *Launch Speed and Range*.

Explain

Revisit the Key Question to give students an opportunity to reflect on their learning experience and verbalize understandings about the science concepts explored in the investigation. Curiosityplace.com resources, including student readings, videos, animations, and whiteboard resources, as well as readings from your current science textbook, are other tools to facilitate student communication about new ideas.



Science Content Video
Average and Instantaneous Speed



Animation
Average and Instantaneous Speed

Elaborate

Investigation B3 offers an excellent opportunity to elaborate on the content through co-instruction with your students' math classes. Coordinate with your math department so that students conduct Investigation B3 at the same time they are learning about graphing functions.

Evaluate

- During the investigation, use the checkpoint ✓ questions as opportunities for ongoing assessment.
- After completing the investigation, have students answer the assessment questions on the *Evaluate* student sheet to check understanding of the concepts presented.

Explore INVESTIGATION B3

Name _____ Date _____

B3 Relating Launch Speed and Range

What function relates the range and launch speed of a projectile?

Materials:
- ✔ Data from Investigation B2
- ✔ Calculator

In the last investigation, you discovered that the range of the marble increases as the launch speed increases. What we would like to learn is exactly *how* the range of the marble changes with the launch speed. For example, how much will the range change if you double the launch speed? In this investigation, you will find a mathematical relationship between the two variables (range and launch speed) that will allow you to predict, and then confirm, the answer.

❶ Finding a mathematical relationship between launch speed and range

For this part of the investigation, you will need your data and graph from Investigation B2, *Launch Speed and Range*.

Identify the highest and lowest launch speeds and their corresponding ranges from your data. Record the speeds and ranges in Table 1. Then, divide the larger values by the smaller ones to find the ratios, and record those in the table.

Table 1: Comparing ratios of launch speeds and ranges

	Launch speed (m/s)	Range (m)
Larger value	8.64	6.95
Smaller value	3.52	1.43
Ratio	2.45:1	4.86:1

a. How do the ratios compare?

The ratio of the ranges is twice as great as the ratio of the speeds.

✓ **b.** What is the approximate mathematical relationship between the two ratios?

It appears that the range is increasing at twice the rate as the launch speed.

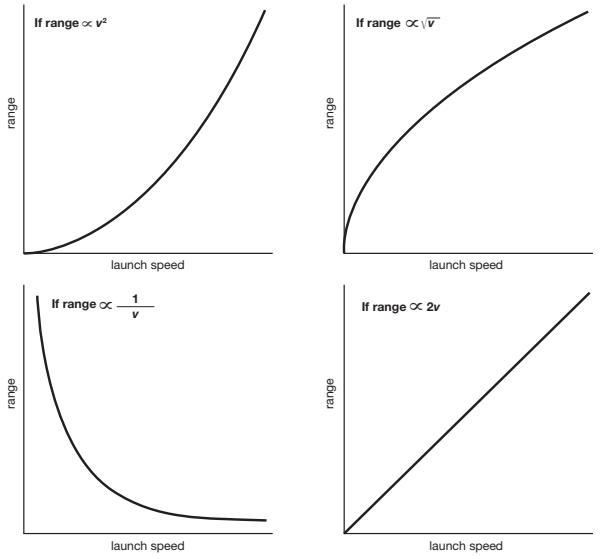
Explore INVESTIGATION B3

❷ What are functions?

You have been investigating the relationship between two variables: launch speed and range. A **function** is a mathematical operation that relates two or more variables. Here is an example of the way a function can be expressed:

$$R \propto \sqrt{v}$$

The symbol "∝" means "is proportional to." In words, the equation above translates to "the range (R) is proportional to the square root ($\sqrt{}$) of the launch speed (v)." The square root is the function that relates the range to the launch speed.



Guiding the INVESTIGATION

❶ Finding a mathematical relationship between launch speed and range

Be sure that your students understand that they must select several lines of data from their B2 data table. Consider copying a set of sample data and distributing to students for use in this investigation. Students may benefit from the ability to underline or write notes on a data table without worrying about losing or obscuring their data.

Explore — INVESTIGATION B3

❸ Finding the function

Looking at the shape of a graph can help you determine the function that relates the variables. Compare the shape of your launch speed vs. range graph from Investigation B2 to the examples shown in Part 2. Choose the graph that is shaped like yours. The function you should use to relate range to launch speed is shown next to each graph.

✓ **a.** Use your graph and the comparison of the ratios to make a prediction about the function that relates the range to the launch speed. Write your function in the parentheses in the last column of Table 2 below.

Based on the shape of our graph and on the ratios of range and launch speed, the function that relates range and launch speed is range ∝ launch speed^2.

b. You can test your prediction by making a graph. If the range is proportional to the chosen function of launch speed, then a graph of the range vs. the function of launch speed should be a straight line. Apply the function to each of the five launch speed values and enter the values in Table 2.

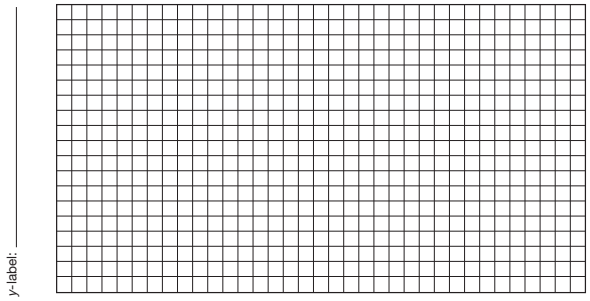
Table 2: Range as a function of speed

Spring setting	Range (m)	Launch speed (m/s)	Function of launch speed (v^2)
1	1.43	3.52	12.4
2	2.39	4.75	22.6
3	3.76	6.13	37.6
4	5.33	7.31	53.4
5	6.95	8.64	74.6

Explore — INVESTIGATION B3

c. Construct a graph of range vs. function of launch speed. Plot the function of launch speed on the *x*-axis and the range on the *y*-axis. Label each axis and include a title on your graph.

Title: _____



x-label: _____

See 3c sample graph.

d. Describe the shape of your graph.

The graph is a straight line.

e. Was your prediction about the function relating range and launch speed correct? Why or why not?

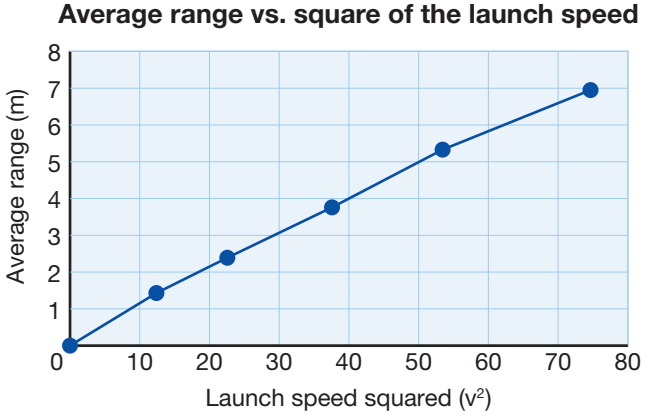
My prediction about the function relating range to launch speed is correct because the graph of the range vs. speed squared is a straight line.

Guiding the INVESTIGATION

❸ Finding the function

In this investigation, students work with sample data having an initial data point at spring setting 1 and continuing through spring setting 5. It is useful to include an initial data point of 0, 0 in the graph, particularly if students are using a spreadsheet to fit a trend line to the data.

3c sample graph



Evaluate

INVESTIGATION B3

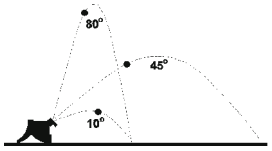
Name _____ Date _____

1. The statement that best represents the relationship between the range of a projectile and its launch speed is (circle the best answer):
   - **a.** "Range is proportional to speed."
   - **b.** "Range is inversely proportional to speed."
   - **c.** "Range is proportional to speed squared."
   - **d.** "Range is proportional to the square root of speed."

2. In this investigation, range was determined by the launch speed of the marble. Which variable is the experimental variable?

   Launch speed is the independent variable in this investigation.

3. Three marble launches are made using the same spring setting. Observers note that the marble has the greatest range at 45 degrees. Does this mean that the launch speed of the marble was greater at 45 degrees than at any other angle? Why or why not?



No, the marble has the same launch speed at 45 degrees as at all other angles since the spring was set to the same notch each time.

Evaluate

INVESTIGATION B3

Name _____ Date _____

4. What does the symbol ∝ indicate in the investigation?

   The symbol ∝ indicates proportionality. It can be read as "is proportional to."

5. Why was it important to make a graph of range vs. the function of launch speed?

   A graph of range vs. the function of launch speed provided a test to determine if the function correctly approximated the relationship between range and launch speed.

SCIENCE AND MATH

Deriving functions from experimental data Deriving mathematical functions from experimental data can be a useful way to approximate how systems will act in the future. However, it is important to emphasize that these functions can only be a mathematical model of actual events. Investigation B3 asks students to reflect on the shape of the graph of their experiment data. Of course, simply because the function is derived correctly does not mean that it will predict the result of launching the marble for any given spring setting. Other factors that are not captured in the experimental data affect each launch. Remind students to reflect on this and to think about the limits of their mathematical models.

WRAPPING UP

Have your students reflect on what they learned from the investigation by answering the following questions:

1. What is a function?
2. What is meant by the term *average range*?
3. What did you learn about the relationship between average range and launch speed in this investigation?

Notes and Reflections