Marble Launcher Investigations

Level A Investigations

A-1 **Launch Angle and Distance**

Which launch angle makes the marble travel the farthest?

Students determine the launch angle that will cause a marble to travel a maximum distance. They discover that there is more than one angle at which the marble may be launched to reach a given distance. This Investigation provides an excellent opportunity to stress the importance of multiple trials when taking data. The basics of graphing are reinforced as students analyze the data.

A-2 **Launch Speed and Distance**

How does launch speed affect the distance traveled by the marble?

Students try to identify the relationship between launch speed and the distance traveled by the marble. A marble is launched at a constant angle at five different launch speed settings. Students graph distance vs. launch speed setting and discover that the graph is not a straight line, but a curve.

Level B Investigations

B-1 **Launch Angle and Range**

Which launch angle will give the marble the best range?

In this Investigation, students find the angle for achieving maximum range with a projectile. Also, a measurement of error is made of their data.

B-2 **Launch Speed and Range**

How does launch speed affect the range of a projectile?

Students try to identify the relationship between launch speed and range. A marble is launched at a constant angle at five different speeds. Students graph range vs. launch speed and discover that the graph is not a straight line, but a curve. In order to determine the exact mathematical relationship between the two variables, they will need to further analyze the data. This occurs in the next Investigation.

B-3 **Relating Launch Speed and Range**

What function relates the changes in range and launch speed of a projectile?

In this Investigation, students identify the function relating projectile range to launch speed. In pursuing this goal, students are exposed to a variety of graphical relationships.

Level C Investigations

C-1 Projectile Motion and the Range Equation

How can you predict the range of a projectile?

In this Investigation, students derive an equation that allows them to predict the range of the marble given the initial velocity and launch angle. First, they derive the range equation, then they test their predictions. Finally, they compare theoretical predictions to actual measurements using a range vs. launch angle graph.

C-2 Improving the Range Equation

How can you improve the range equation?

In this Investigation, students correct the inconsistencies in the range equation they derived previously. They discover that the differences between theory and actual measurements are due to the fact that the marble is actually launched above the floor and the range equation assumes that it is launched at floor level. They use a quadratic equation to correct for this height difference.

C-3 Accuracy, Precision, and Error

How repeatable are experiments with the marble launcher?

Students determine the accuracy and precision of the marble launcher experiments. They launch the marble ten times at the same launch angle and initial velocity onto a sheet of grid paper covered by carbon paper. From this, they create a target plot that shows the average deviation of all landings from the average. As a challenge, students apply what they learned by playing "shoot your own grade" using a target they create.

Question: How can you predict the range of a launched marble?
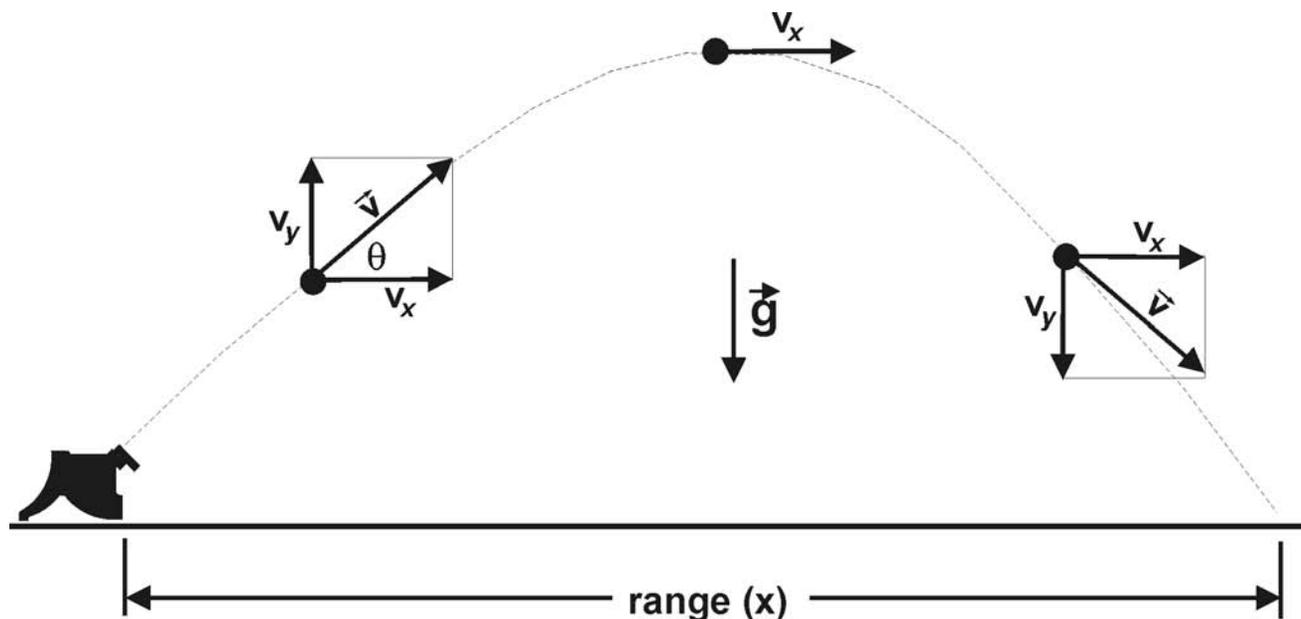
In this Investigation, you will find and test a model that will predict the range of the marble from the initial velocity and launch angle.

You have learned that the motion of any object moving through the air affected only by gravity is an example of **projectile motion**. Examples of projectile motion include a basketball thrown toward a hoop, a car driven off a cliff by a stunt person, and a marble launched from the CPO marble launcher. Projectile motion is also called two-dimensional motion because it depends on two components: vertical and horizontal. In this Investigation, you will determine a mathematical model (the **range equation**) that predicts the range of the marble given launch angle and initial velocity.

1   Analyzing the motion of the marble in two dimensions

How can you predict the range of the marble? Since gravity pulls *down* and not sideways, the motion of the marble must be separated into *components*. It makes sense to pick one component (y) in the vertical direction aligned with gravity. The other component (x) is then chosen to be in the horizontal direction, *perpendicular* to the force of gravity. The diagram below shows the velocity of the marble (v) at three points in its trajectory, resolved into x and y components, v_x and v_y.


a. Use the diagram above to explain why projectiles travel in a curved path called a *trajectory.*

b. How does the marble's velocity in x change over the time of the flight? How does its velocity in y change over the time of flight?

The object of this Investigation is to find and test a model that will predict the range of the marble from the initial velocity and launch angle—the range equation.

a. The first step is to separate the velocity of the marble into x and y components. Use the triangle formed by velocities (at right) to express v_x and v_y in terms of the initial velocity, v, and the sine and cosine of the launch angle, θ.
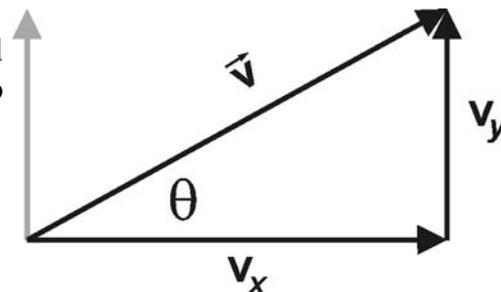
When the initial velocity is separated into x and y components, Equations 1a-2b give the relationships between the motion variables *separately* for x and y. In these equations the subscript i refers to the *initial* values at launch.

Equations 1a and 1b are for the marble's velocity while equations 2a and 2b are for the marble's position.

Equation 1a $$v_x = v_{xi} + a_x t$$

Equation 1b $$v_y = v_{yi} + a_y t$$

Equation 2a $$x = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Equation 2b $$y = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

b. Since gravity does not pull up or sideways, one of the accelerations (a_x, a_y) is -g, and the other is zero. For the first approximation, the range (x) is defined so the initial x and y positions are also zero. Rewrite Equations 1a-2b leaving out terms that are zero and substituting your previous results for v_{xi} and v_{yi}.

Equation 1a: _____

Equation 1b: _____

Equation 2a: _____

Equation 2b: _____

c. The purpose of this exercise is to find a theory that predicts where the marble will land (the range, x) given the initial velocity and launch angle. This problem can be solved in several steps. First, assume that gravity acts only on the y component of velocity. Solve for the *time* it takes the marble to reach its maximum height (where $v_y = 0$).

d. Since gravity does not pull sideways, the x component of the marble's velocity is not affected and remains constant. Use Equation 2a to calculate the range (x) from v_{xi} and the total time of flight (this will be the range equation).
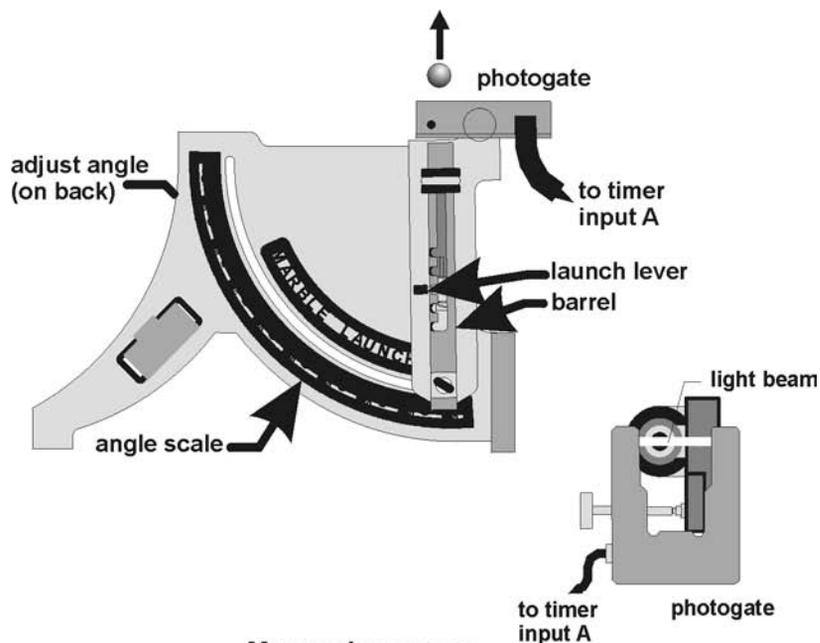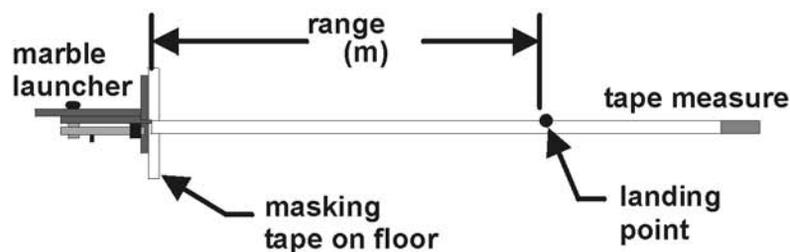
1. Attach one photogate to the marble launcher so that the marble breaks the light beam as it comes out of the barrel. Put the timer in interval mode, and connect the photogate to input A. The launcher can launch at angles from 0 (horizontal) to 90 degrees (vertical). For this experiment, you will use angles from 0 to 80 degrees.

   The photogate attaches to the tab on the end of the wood piece that supports the barrel. Be sure the light beam crosses the center of the barrel.

2. Use a strip of masking tape on the floor to make sure that the marble launcher is consistently placed in the same location. A tape measure laid along the floor provides a good range reference.

3. To launch marbles, pull the launching lever back and slip it sideways into one of the slots. Put a marble in the end of the barrel and the marble launcher is ready to launch.

4. There are five notches that change the compression on the spring and give different launch speeds. In this experiment, you will change the launch speed setting for different launches.

photogate

adjust angle
(on back)

to timer
input A

launch lever
barrel

light beam

angle scale

to timer
input A

photogate

Measuring range

range
(m)

marble
launcher

tape measure

masking
tape on floor

landing
point

Pull the lever
back and slip
it into one of
the five notches

Launch the marble by
flicking the lever to the
center

The marble
should sit
loosely in
the end

5 4 3 2 1

There are 5 launch
speed settings
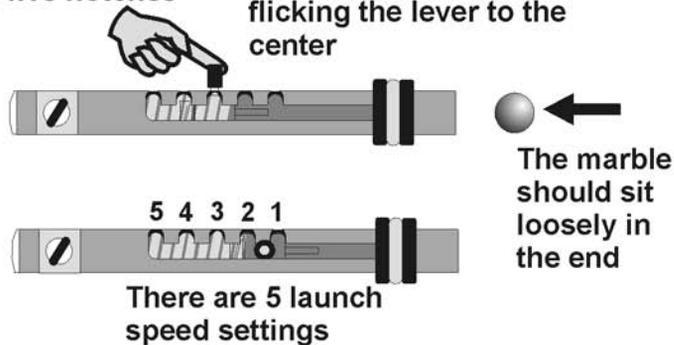
SAFETY RULES:

- **Never launch marbles at people.**
- **Wear safety glasses or other eye protection when launching marbles.**
- **Launch only the black plastic marbles that come with the marble launcher.**

4 · Doing the experiment

1. To check your theory, a spread of data for different *launch angles* and *initial velocities* will be needed. For each spring setting you should have five different launch angles from 20 degrees to 80 degrees. To cover both variables (angle and speed), at least 20 data points are needed.
2. A minimum of two people are needed per launcher. One person launches the marbles and the other person/people watches where they land.
3. Use *only* the black plastic marbles provided by CPO.
4. Record the spring setting, launch angle, time from photogate A, and measured range for each launch. It often takes several launches with the same setup to locate the landing point precisely. For each setup, you may need to run several trials until the measured range is consistent within 5 cm.
5. Calculate the initial velocity by dividing the distance traveled (width of marble = 0.019 m) by the time at photogate A. Record initial velocities in the table. *A larger version of the table is found on your answer sheet*.

Spring setting (1 - 5):

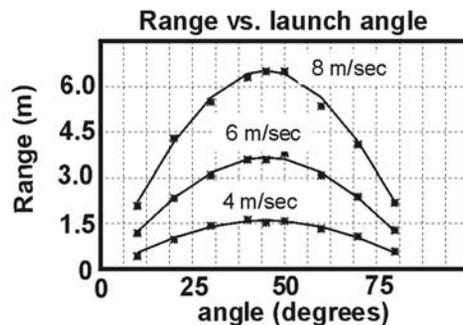| Launch angle (degrees)	Range (m)	Distance (m)	Time from A (sec)	Initial velocity (m/sec)
		0.019		
		0.019		
		0.019		
		0.019		
		0.019		

5 · Comparing theory predictions to measured data

Use the table to help you compare your measured data to theory predictions using your range equation. Fill in several launch angles for each initial velocity. *A larger table is found on your answer sheet*.

Initial velocity (m/sec)		Launch angles (degrees)					
	x (predicted)						
	x (measured)						

a. Make a graph showing the range vs. launch angle for several different initial velocities. The graph at right is one example of how this graph could look. You could also choose other ways to graph the data. Plot the measured points as unconnected dots and the theoretical values as solid lines since the theory predicts the speed of the marble at all points.

Range vs. launch angle — Range (m): 6.0, 4.5, 3.0, 1.5, 0; angle (degrees): 0, 25, 50, 75; curves labeled 8 m/sec, 6 m/sec, 4 m/sec.

b. How does your theory compare with your measurements? In particular, is there a consistent deviation between theory and experiment? The word "consistent" means the difference between the theoretical and experimental data seems to depend on something in the experiment and is not random. For example, a consistent deviation would occur if the measured range for small angles is *always* smaller than predicted by theory *regardless of the velocity*. Explain how the consistent deviations you found are affected by velocity and angle.

c. Consistent deviations indicate that something is missing from the theory. What is missing, and why does it have the observed effect on your results?

Name:

C-1   Projectile Motion and the Range Equation

Question: How can you predict the range of a launched marble?

1   Analyzing the motion of the marble in two dimensions

a. Use the diagram to explain why projectiles travel in a curved path called a *trajectory*.

_____

_____

b. How does the marble's velocity in *x* change over the time of the flight? How does its velocity in *y* change over the time of flight?

_____

_____

2   Understanding the velocity equations

a. Use the triangle formed by velocities to express v_x and v_y in terms of the initial velocity, *v*, and the sine and cosine of the launch angle, θ.

_____

_____

_____

b. Rewrite Equations 1a-2b leaving out terms that are zero and substituting your previous results for v_{xi} and v_{yi}.

   Equation 1a:
_____

   Equation 1b:
_____

   Equation 2a:
_____

   Equation 2b:
_____

c. The purpose of this exercise is to find a theory that predicts where the marble will land (the range, *r*) given the initial velocity and launch angle. This problem can be solved in several steps. First, assume that gravity acts only on the y component of velocity. Solve for the *time* it takes the marble to reach its maximum height (where $v_y = 0$).

_____

_____

d. Since gravity does not pull sideways, the *x* component of the marble's velocity is not affected and remains constant. Use Equation 2a to calculate the range (*r*) from v_{xi} and the total time of flight (this will be the range equation).

_____

_____

3 | **Setting up the marble launcher**

4 | **Doing the experiment**

Spring setting (1 - 5):

Launch angle (degrees)	Range (m)	Distance (m)	Time from A (sec)	Initial velocity (m/sec)
		0.019		
		0.019		
		0.019		
		0.019		
		0.019		

Spring setting (1 - 5):

Launch angle (degrees)	Range (m)	Distance (m)	Time from A (sec)	Initial velocity (m/sec)
		0.019		
		0.019		
		0.019		
		0.019		
		0.019		

C-1 Projectile Motion and the Range Equation   **Answer Sheet**

Spring setting (1 - 5):

Launch angle (degrees)	Range (m)	Distance (m)	Time from A (sec)	Initial velocity (m/sec)
		0.019		
		0.019		
		0.019		
		0.019		
		0.019		

Spring setting (1 - 5):

Launch angle (degrees)	Range (m)	Distance (m)	Time from A (sec)	Initial velocity (m/sec)
		0.019		
		0.019		
		0.019		
		0.019		
		0.019		

Comparing theory predictions to measured data

Use the table to help you compare your measured data to theory predictions using your range equation. Fill in several launch angles for each initial velocity.

Initial velocity		Launch angles (degrees)					
(m/sec)							
	x (predicted)						
	x (measured)						
	x (predicted)						
	x (measured)						
	x (predicted)						
	x (measured)						
	x (predicted)						
	x (measured)						
	x (predicted)						
	x (measured)						

a. Make a graph showing the range vs. launch angle for several different initial velocities. The graph at right is one example of how this graph could look. You could also choose other ways to graph the data. Plot the measured points as unconnected dots and the theoretical values as solid lines since the theory predicts the speed of the marble at all points.

Title: _____

y label: _____

x label: _____

b. How does your theory compare with your measurements? In particular, is there a consistent deviation between theory and experiment? The word "consistent" means the difference between the theoretical and experimental data seems to depend on something in the experiment and is not random. For example, a consistent deviation would occur if the measured range for small angles is *always* smaller than predicted by theory *regardless of the velocity*. Explain how the consistent deviations you found are affected by velocity and angle.

_____

_____

_____

c. Consistent deviations indicate that something is missing from the theory. What is missing, and why does it have the observed effect on your results?

_____

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Questions

1. A marble launcher is set up on the floor using notch 3 and an angle of 45 degrees. The marble is launched and the range is measured. The launcher is then taken off the floor and put on a nearby table. It is again set for notch 3 and 45 degrees. Will the two flights be the same or different? Explain your answer.

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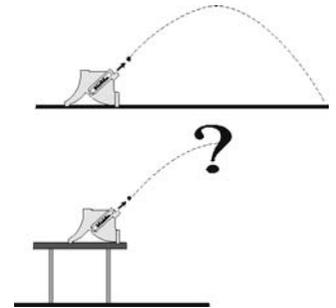
2. Sketch the vector triangles showing the initial velocity and the x and y components for each of the three launch angles shown. Using your diagrams, explain why the marble launched at 45 degrees goes the farthest.

80°

45°

10°
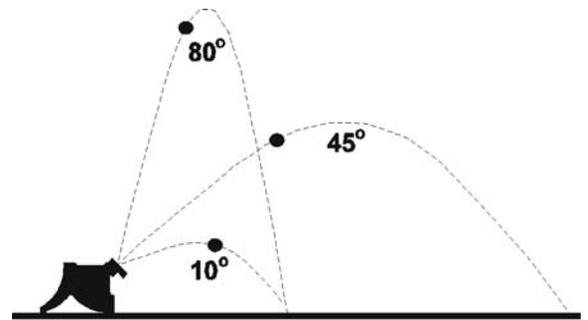
3. Where along the trajectory is the vertical component of the velocity of the marble zero? Circle this place on the marble's path.

Where is $V_Y = 0$?
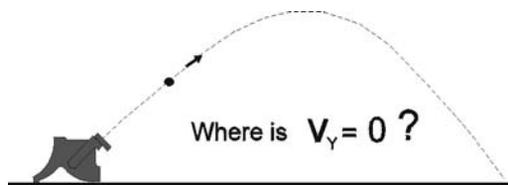
4. The marble launcher is set up with an angle of 80 degrees, and the timer measures 0.0030 seconds for the 0.019 m marble. Calculate the expected range (*x*) of the marble.

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C-1 Projectile Motion and the Range Equation   **Answer Sheet**

Curriculum Resource Guide: Marble Launcher

Credits

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